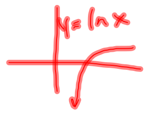
24. $\lim_{x\to 0^+} (\sin x)^x = y \quad 0^0$

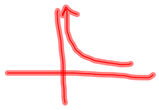
$\lim_{x\to 0^+} \ln(\sin x)^x = \ln y$

$\lim_{x\to 0^+} x\ln(\sin x) = \ln y$

$\frac{-\infty}{\infty} \quad \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} = \ln y$



$\lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot \cos x}{-\frac{1}{x^2}} = \ln y$



$\frac{0}{0} \quad \lim_{x\to 0^+} -\frac{\cos x \cdot x^2}{\sin x} = \ln y$

$\lim_{x\to 0^+} \frac{-\cos x\cdot 2x + x^2\sin x}{\cos x} = 0 = \ln y$

$\boxed{1} = e^0 = y$

Feb 2-12:39 PM

8.3 relative rates of growth

which functions grow faster (or slower)

If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$ then f grows faster

If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$ then g grows faster

if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$, $0 < L < \infty$, same

Feb 2-12:48 PM

Compare $y = x+1$ with $y = x^2-3$

$\frac{\infty}{\infty}$ $\lim_{x\to\infty} \frac{x^2-3}{x+1} = \lim_{x\to\infty} \frac{2x}{1} = \infty$

so x^2-3 grows faster

Compare $y = x^3$ with $y = 2x^3$

$\lim_{x\to\infty} \frac{x^3}{2x^3} = \frac{1}{2}$ same

Feb 2-12:54 PM

$y = \sqrt{x^2+5}$

$y = (2\sqrt{x}-3)^2$ > compare with $y = x$

$\cancel{\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{(2\sqrt{x}-3)^2}}$

$\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{x} = \lim_{x\to\infty} \frac{2x\frac{1}{2}(x^2+5)^{-\frac{1}{2}}}{1}$

$\cancel{\lim_{x\to\infty} \frac{x}{\sqrt{x^2+5}}}$

$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2}} = \lim_{x\to\infty} \sqrt{\frac{x^2+5}{x^2}}$

$\lim_{x\to\infty} \sqrt{1+\frac{5}{x^2}} = 1$

$\lim_{x\to\infty} \frac{(2\sqrt{x}-3)^2}{x}$

$\lim_{x\to\infty} \frac{(2\sqrt{x}-3)^2}{(\sqrt{x})^2}$

$\lim_{x\to\infty} \left(\frac{2\sqrt{x}-3}{\sqrt{x}}\right)^2$

$\lim_{x\to\infty} \left(\frac{2\sqrt{x}}{\sqrt{x}} - \frac{3}{\sqrt{x}}\right)^2 = 4$

both grow at same rate as $y=x$

Feb 2-1:02 PM